Fermion helicity flip by parity violating torsion

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The helicity flip of a spin-1/2 Dirac fermion interacting with a torsion- field endowed with a pseudo-tensorial extension is analysed. Taking the torsion to be represented by a Kalb-Ramond field, we show that there is a finite amplitude for helicity flip for massive fermions. The lowest order contribution which turns out to be proportional to the pseudo-tensor term, implies a new physical understanding of the phenomenon of helicity flip.

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In the past few years, research in the field of space-time with torsion has yielded a number of interesting results. In particular, the extension of the Dirac-Einstein Lagrangian by incorporating a Cartan term, has been studied extensively. This is due to the fact that the spin of a particle turns out to be related to torsion just as mass is responsible for curvature [1]. The possibility for a flip in helicity which is of enormous interest in the context of the early universe and solar neutrinos, has been explored [2–4]. It has been proved that in the presence of torsion, helicity is not conserved [2].

One very important observation is that the presence of torsion destroys the cyclic property of the Riemann-Christoffel tensor. As a result, the standard Einstein-Hilbert action admits a parity-violating term [5]. Since the very presence of torsion automatically allows parity-violation in the Lagrangian for pure gravity, it should be possible to incorporate the latter in the geometry of space-time itself. In a previous paper [6], SS in collaboration with Mukhopadhyaya showed that this could be achieved by suitably extending the covariant derivative with a set of pseudo-tensorial connections, proportional to the torsion tensor itself. This gave a unique prescription in terms of the covariant derivatives to obtain parity-violating effects in the Lagrangians for pure gravity as well as for matter fields with arbitrary spin. Such a theory not only allows one to couple electromagnetic field with torsion in a gauge invariant way but is also supported by string theory [7].

Here we show that a space-time dependent torsion, endowed with a pseudo-tensorial extension can produce a flip. The flipping amplitude for our process turns out to be dependent solely on the extension and vanishes had the extension not been there. In case of massive fermions where a chirality state is a linear superposition of two helicity states and vice versa, the helicity flip obviously results in chirality flip as well. Throughout the calculation we take a quantized massive fermion field in a classical Kalb-Ramond background. The motivation of this work is to find an alternative prescription which may explain the helicity flip of a massive fermion from a physical standpoint so far unexplored. For a non-vanishing neutrino mass this result may turn out to be significant in the context of the solar neutrino problem.

The most general connection in space-time geometry includes, in addition to the symmetric and antisymmetric parts, a pseudo-tensorial part as well. Such a pseudo-tensorial extension can lead to parity violation in gravity. It was shown [6] that the most general affine connection is:

$$\tilde{\Gamma}^{\kappa}_{\nu\lambda} = \Gamma^{\kappa}_{\nu\lambda} - H^{\kappa}_{\nu\lambda} - q(\epsilon^{\gamma\delta}_{\nu\lambda} H^{\kappa}_{\gamma\delta} - \epsilon^{\kappa\alpha}_{\beta\lambda} H^{\beta}_{\gamma\alpha} + \epsilon^{\kappa\alpha}_{\beta\nu} H^{\beta}_{\lambda\alpha}) \tag{1}$$

Here Γ's are the usual Christoffel symbols, H is the contorsion tensor, q is parameter determining the extent of parity violation, depending, presumably on the matter and spin distribution and so can be helicity dependent. It has been shown [7] that the torsion tensor H can be equated to the field strength of the background second rank massless antisymmetric tensor field namely the Kalb-Ramond field which appears in the massless sector of the string multiplet.

Using (1) to extend the Dirac-Einstein Lagrangian for a spin-1/2 field [8,9] in a spacetime with torsion, we get:

$$\mathcal{L}_{Dirac} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} - \sigma^{\rho\beta} v^{\nu}_{\rho} g_{\lambda\nu} \partial_{\mu} v^{\lambda}_{\beta} - g_{\alpha\delta}\sigma^{ab} v^{\beta}_{a} v^{\delta}_{b} \tilde{\Gamma}^{\alpha}_{\mu\beta})]\psi \tag{2}$$

where one has introduced the tetrad v^{μ}_{a} to connect the curved space with the corresponding tangent space at any point and σ^{ab} is the commutator of the Dirac matrices. The Greek indices correspond to the curved space and the Latin indices to the tangent space. Using the full form of $\tilde{\Gamma}$ defined in (1) , $\mathcal{L}_{Dirac}$ can be expressed as

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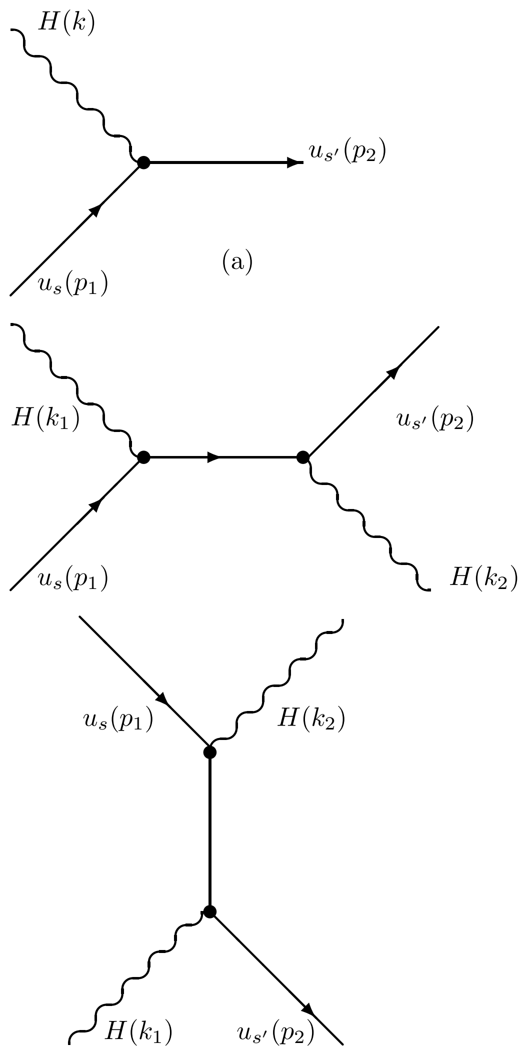
$$\mathcal{L}_{Dirac} = \mathcal{L}^E + \bar{\psi}[i\gamma^\mu g_{\alpha\delta}\sigma^{\beta\delta}H^\alpha_{\mu\beta}]\psi + \mathcal{L}^{pv} \tag{3}$$

The first term is the one obtained in Einstein gravity, the second corresponds to the Cartan extension($\mathcal{L}^C$). The third term results from the pseudo-tensorial extension of the affine connection and is responsible for parity violation. This term is given later explicitly in equation (5).

$\mathcal{L}_{Dirac}$ is used to calculate the transition amplitude from one helicity state to another. By helicity states we mean the spinors corresponding to the eigen-states of the operator $\Sigma.\hat{p}$. Presumably one could work also with the chirality states, the spinors corresponding to the eigen-states of γ_5. In the presence of torsion, helicity is not conserved [2] and as such its flipping is considered. To calculate the flip we remember that the field H being a classical field can only appear as the external lines in the Feynman diagrams. We therefore consider the following lower order Feynman diagrams:



(b)

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Figure 1: Leading order Feynman diagrams for helicity flip

For a weak gravitational field, we set $g_{\alpha\beta} = \eta_{\alpha\beta}$ where $\eta_{\alpha\beta}$ is the Minkowski metric diag(1,-1,-1,-1). H cannot be a constant since it implies that the field H carries no energy or momentum, which cannot be possible. As was argued in [7], H can be written as the Hodge-dual to the derivative of a massless pseudo-scalar field $\tilde{\phi}$, which is the axion of the corresponding String theory.We then get:

$$\mathcal{L}^C = i\bar{\psi}\gamma^\mu\sigma^{\beta\phi}\epsilon_{\phi\mu\beta\lambda}(\partial^\lambda\tilde{\phi})\psi \tag{4}$$

$$\mathcal{L}^{pv} = q\bar{\psi}\gamma^{\alpha}\sigma^{\gamma\nu}\gamma_5\epsilon_{\alpha\gamma\nu\lambda}(\partial^{\lambda}\tilde{\phi})\psi \tag{5}$$

$\mathcal{L}^{\mathcal{E}}$ doesn't contribute in flat space. As such, the calculations are done using $\mathcal{L}_{int} = \mathcal{L}^{C} + \mathcal{L}^{pv}$.

It may be noted that both the parity violating and parity conserving interaction Lagrangians are dimension five operators leading to the usual problem of renormalizability in a full quantum theory. However, here we are considering a semi-classical theory and therefore have the leading order finite amplitude diagrams as drawn above.

Now we first consider that the field H is spatially homogeneous and depends only on time. For an isotropic and homogeneous background geometry this is a valid assumption.

With such a choice for H, if we now consider the Feynman diagram of figure 1(a), we get a factor $\delta^4(p_1 - p_2 + k) = \delta^3(\mathbf{p_1} - \mathbf{p_2} + \mathbf{k})\delta(p_1^0 - p_2^0 + k^0)$ in the S matrix element. This simplifies to a proportionality to $\delta(k^0)$ where k^0 corresponds to the energy of the H field. This will contribute only if k^0 is zero. However since there is only a t-dependence, k^0 cannot be zero since in that case E=0 which means that the field H vanishes identically. Since 1(a) is the fundamental vertex, it is clear that there cannot be a helicity flip. As such a pure time-dependence of H *cannot* produce a helicity flip.

Next we consider that H depends on both space and time. A similar argument as above shows that 1(a) contributes and as such there can be a helicity flip for such an H. The transition-matrix element between s-helicity state to s'-helicity state, in accordance with the Feynman diagram of figure 1(a), is:

$$\mathcal{M}_{if} = 6\left[\bar{u}_{s'}(q + \gamma_5)\not{k}u_s\right] \tag{6}$$

where $\not{k} = \gamma^{\mu}k_{\mu}$.

We use the gamma matrices and spinors in the Dirac-Pauli representation [10].

Assuming the fermion to be initially moving in the z-direction, the result for $+\frac{1}{2}$ helicity to $-\frac{1}{2}$ helicity or vice versa is:

$$\mathcal{M}_{if} = 12k_x m\sqrt{\frac{E+m}{E'+m}} \tag{7}$$

where $E' = E + k^0$.

Note that this amplitude is independent of the pseudo-tensor term. It also suggests that the field H has to have a non-zero momentum component in a direction that is different from the fermion direction. In fact, if in the lab frame, the momentum of the field H points in the same direction as that of the fermion, there cannot be a helicity flip. The transition amplitude is proportional to mass, but it is modified by the presence of k_x. However this cannot be the lowest order contribution since it is kinematically forbidden. This can be seen by noting that the energy-momentum relations and the mass constraints cannot be satisfied simultaneously.

Now let's consider the next order Feynman diagrams in fig 1b. The second diagram in 1b is analogous to the crossed diagram that is obtained for Compton scattering. The transition-matrix element between s-helicity state to s'-helicity state, in accordance with the Feynman diagram of figure 1(b), is:

$$\mathcal{M}_{if} = \left[36\bar{u}_{s'}(q' + \gamma_5)\not{k}_2\frac{\not{p}_1 + \not{k}_1 + m}{(p_1+k_1)^2 - m^2}(q+\gamma_5)\not{k}_1 u_s\right] \tag{8}$$

For the 2nd diagram in figure 1(b), we get:

$$\mathcal{M}_{if} = \left[36\bar{u}_{s'}(q' + \gamma_5)\not{k}_1\frac{\not{p}_1 - \not{k}_2 + m}{(p_1-k_2)^2 - m^2}(q+\gamma_5)\not{k}_2 u_s\right] \tag{9}$$

We use $q' = -q$. In doing so, we assume that q is helicity-dependent. The inspiration behind this is as follows. In the matrix element (8), there are terms like $(q+\gamma_5)$. It is well known that for highly energetic fermions, $(1+\gamma_5)$ seeks out the right-handed particles while $(1-\gamma_5)$ seeks out the left-handed particles. In order to get terms resembling these, it is necessary to make q helicity-dependent. If we don't make such a choice we get other transitions which are briefly mentioned later.

Let's consider eqn(8). Put $p_1 + k_1 = K$. From here, the calculation can be simplified. Writing $\not{k}_1 = k_1 \cdot \gamma$ and applying the Dirac equation, we get

$$\not{k}_1 u_s = (\gamma.K - m)u_s \tag{10}$$

Similarly,

$$\bar{u}_{s'} \not{k}_2 = \bar{u}_{s'}(\gamma.K - m) \tag{11}$$

Using,

$$(\gamma.K - m)(\gamma.K + m) = K^2 + m^2 \tag{12}$$

and analogous relations for the 2nd diagram in figure 1(b), it is now easy to verify that the Feynman diagrams of fig 1b generate the following result:

$$\mathcal{M}_{if} = 144mq\bar{u}_{s'}(p_2)\gamma_5 u_s(p_1) \tag{13}$$

This shows that the Einstein-Cartan contribution to the matrix element vanishes and we are left with a contribution proportional to q. *Thus only the pseudo-tensor extension can produce a flipping.* Also the flipping amplitude is proportional to mass. Curiously enough, the only dependence on the H field is in the factor q.

We evaluate (13) for a particular case. Suppose the interaction is confined in the x-z plane. Let the z-axis be aligned with the initial direction of the fermion before it interacts. Using the Dirac-Pauli representation as in [10], the following result is obtained:

$$\mathcal{M}_{if} = 144qmp_{2x}\sqrt{\frac{E+m}{E_2+m}} \tag{14}$$

Here subscript 2 refers to the fermion after interaction. The amplitude is for right-handed helicity to left-handed helicity transition or vice-versa. Obviously the flipping amplitude vanishes for zero-mass fermions.

We have used a Dirac-Einstein Lagrangian modified by torsion and a pseudo-tensorial extension, to calculate the flipping between helicity states for massive fermions. It was shown that for a specific process, the flipping probability is non-zero for massive fermions and solely depends on the extension. The crucial assumption that we have made is that the coupling of the extension term can be helicity dependent. Presumably it behaves something like the electric charge and is equal and opposite for opposite helicities. This assumption makes the Feynman rules for our process resemble the V-A rules in Weak Interaction. It is important to note that while one gets a nonvanishing flipping probability from the fermion mass itself, here we find an additional contribution from the pseudo tensorial extension of the torsion. This extra term contains the coupling q.

It is interesting to note that even if q were to be independent of helicity, we get a finite amplitude for helicity flipping for particle to anti-particle transitions. Such transitions are probably allowed for our Lagrangian since it doesn't conserve **C**. Such analysis can form the basis of future ventures in this direction.

Our work thus opens up a completely new physical possibility, namely parity violating torsion, to explain the spin flip of massive fermions. Whether the fermions couple to the background in the way discussed here, depends on the type of theoretical model we consider. However in a string theory inspired model where we indeed have a Kalb-Ramond background, such a coupling occurs naturally. In this context we propose that the result of this work can be a possible explanation for the so called solar neutrino problem where the number of left handed neutrino detected is less than that expected. For a quantitative estimation one needs to know the exact nature of the torsion field and its parity violating extension inside the sun. Although we already have a bound on the classical axion, the undetermined coupling parameter 'q' appearing in our theory may be determined from known experimental results. As the contribution of the helicity flip from the neutrino mass, which is very small, fails to explain the observed shortage of left handed neutrino, this work proposes an additional effect in terms of the torsion coupling 'q' as a possible explanation to this longstanding problem. Work in this direction is in progress and will be reported elsewhere.

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